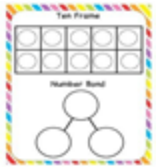
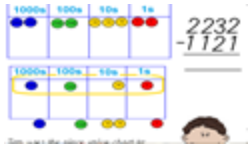
St Michael's
C.E. Primary School

A CPA Approach to Maths Calculations

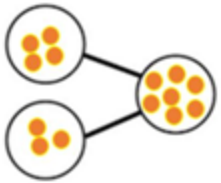
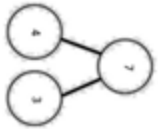
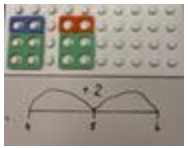
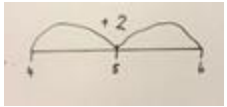
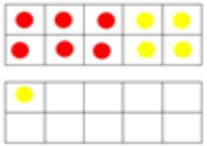
The National Curriculum for Mathematics aims to ensure that all pupils become fluent, develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately; reason mathematically by following a line of enquiry and can solve problems by applying their mathematics in a variety of ways.

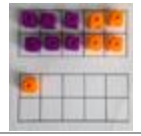
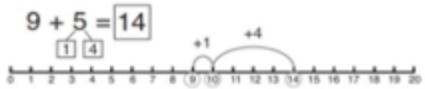
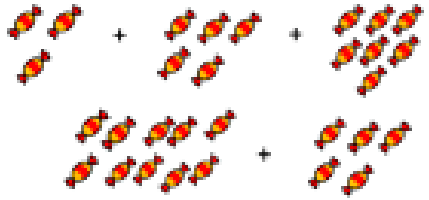
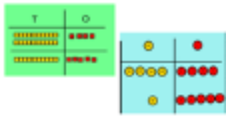
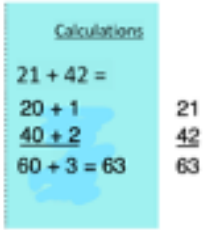
At St Michael's our children will achieve these outcomes through the CPA approach to Maths.

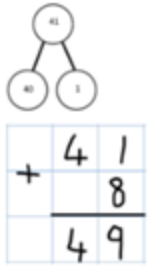
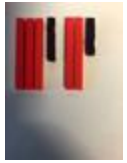
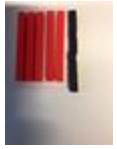
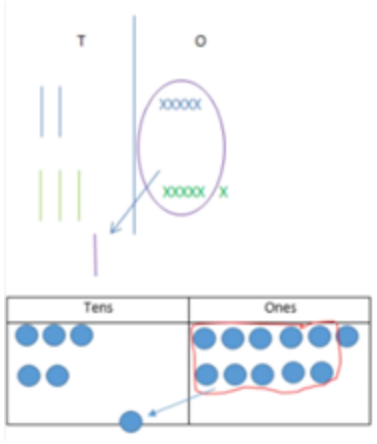
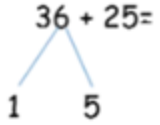
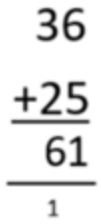
Place Value - Key language which should be used: the value of each digit in whole and decimal numbers

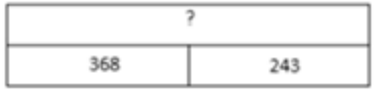
Concrete	Pictorial	Abstract
Number bonds to 10. In depth understanding of each number. For example - 5 is made of 4 and 1 or 5 is made of 3 and 2. 'Friends of 10' - Use of the 10 frame - for example the number 8 best friend = 2 / the number 6 best fried = 4 / the number 7 best friend is 3.		For example - 3 + 2 = 5 / 4 + 1 = 5 / 5 - 3 = 2 / 5 - ? = 3
Place Value chart. For example - 7430 - 7 lots of 1000 / 74 lots of 100 / 743 lots of 10 / 7430 lots of 1 (not zero ones) The use of Place Value Chart to multiply and divide by multiples of 10, 100, 1000.		For example - 7000 + 400 + 30 = 5000 + 2000 + 200 + 200 + 30 =
The use of Ten Frame / Place Value Chart to show the value of digits in decimal numbers.	1 full egg box 8/10) / 1 in another box = 5/10 (DRAW concrete representation)  	For example - 0.8 + 0.5 = 1.3 For example - 0.8 + 0.6 + 0.7 + 0.8 +0.6 = 0.5 + 0.5 + 0.5 + 0.5 + 0.5 0.3 + 0.1 + 0.2 + 0.3 + 0.1
The use of dienes/straws to show the value of each digit within a number.		For example - 10+3 = 13 100+10+6=116

Addition - Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'

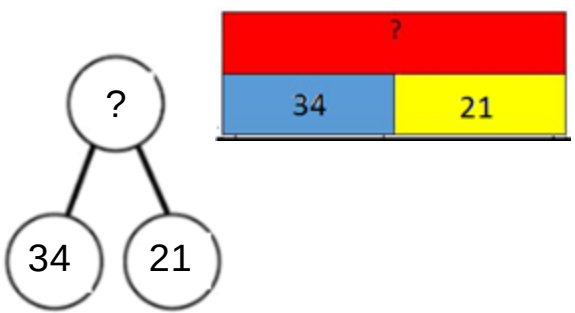
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. egg, shells, teddy bears etc) Egg Boxes - chd know its eight 1's / one five and three ones / two away from ten.   	 Bar Model 	4 + 3 = 7/ 3 + 4 = 7(four is part, three is part and the whole is seven) Including inverse 7 - 4 = 3/ 7 - 3 = 4 
Counting on using number line by using cubes or numicon  	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 2? What's the total of 4 and 2? 4 + 2 - Start from 4, count on 2 
Regrouping to make 10 by using ten frames and counters/cubes or using numicon: 6 + 5 How else can you make 11? 8 + 3, 7 + 4, 9 + 2  	Children to fill in the ten frame templates to fill in with dots. 	Children to develop an understanding of equality e.g 6 + ___ = 11 and __ 6 + 5 = 5 + ___/6 + 5 = __ + 4

Concrete	Pictorial	Abstract
Regrouping to make 10 Start with the bigger number and use the smaller number to make 10.  **6+5=11** 	Use pictures or a number line. Regroup of partition the smaller number to make 10.  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits 4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	Add together three groups of objects. Draw a picture to recombine the groups to make 10. 	Combine the two numbers that make 10 and then add on the remainder. 
Column method – no regrouping 24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	

Concrete	Pictorial	Abstract
TO + O using numicon shapes. Continue to develop understanding of partitioning and place value. 41 + 8   	Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones. 	 41 + 8 8 + 1 = 9 40 + 9 = 49
TO + TO using numicon shapes Continue to develop. This could be done one of two ways: understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25     	This could be done one of two ways: 	Looking for ways to make 10  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 

Concrete	Pictorial	Abstract
Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practise with this they should be able to apply it to larger numbers and the abstract 	Children to represent the counters e.g. like the image below  If the children are completing a word problem, draw a bar model to represent what it's asking them to do 	$$\begin{array}{r} 368 \\ +243 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Fluency variation, different ways to ask children to solve 34 + 21:



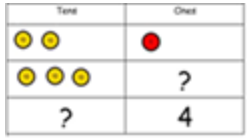
Same saved £34 one week an £21 another, How much did he save in total?

34 + 21 = 55. Prove it! (reasoning but the children need to be fluent in representing this)

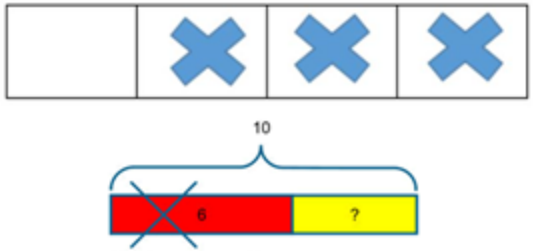
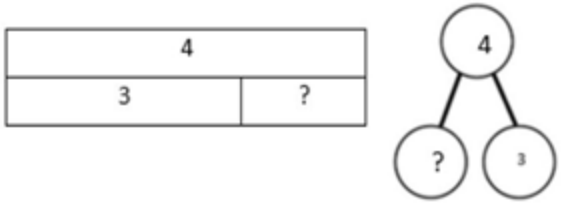
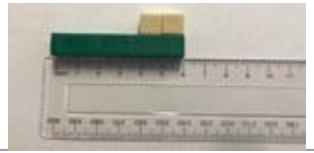
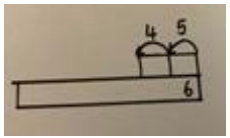
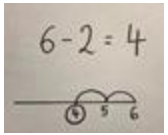
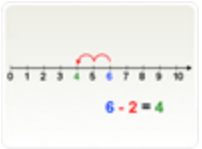
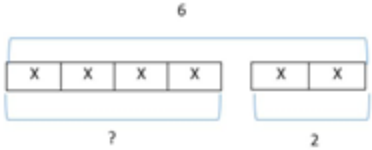
$$\begin{array}{r} 34 \\ +\ 21 \\ \hline \\ \hline \end{array}$$

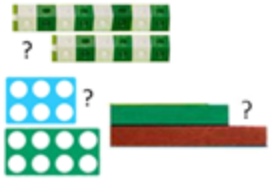
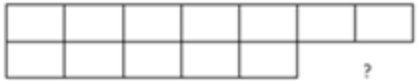
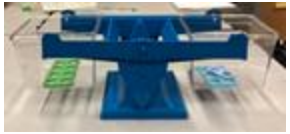
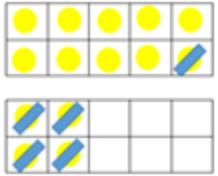
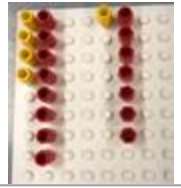
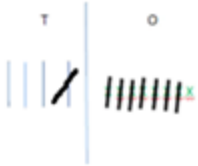
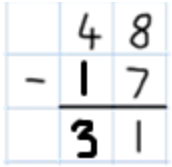
21 + 34 =
____= 21 + 34
What's the sum of twenty one and thirty four?

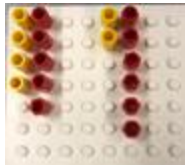
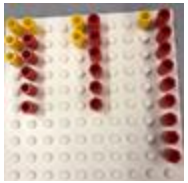
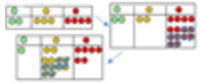
Always use missing digit problems too:



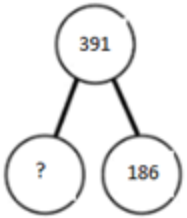
Subtraction - Key language which should be used: less than, the difference, subtract, minus, fewer, decrease, the difference is four.'

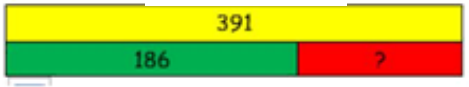
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the objects. 4 - 3 = 1 	Use of the bar model 	4 - 3 = 1 or ___ = 4 - 3 
Counting back to subtract. 	  	Children count back using a number line.  
Counting on to find the difference(using number lines or number tracks) 	Children represent what they pictorially see e.g. 	

Concrete	Pictorial	Abstract
Finding the difference (using cubes, numicon or numicon rods, other objects can also be used) 	Children to draw the cubes/other objects they have used XXXXXXXX XXXXXX Use of the bar model 	Find the difference between 8 and 6. 8 – 6, the difference is ? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)  
Making 10 (using numicon or ten frames)  Children could also do this by subtracting 5 from 10. 	Children to present the ten frame pictorially 	14 – 5 = 9 You also want children to see related facts e.g. 14 – 9 = 5 Children to represent how they have solved it e.g. (crossing tens) 14 - 5 = 9 14 - 9 = 5 9 + 5 = 14 5 + 9 = 14
Column method (using base 10 and numicon) 48 - 17   		48 - 17 = 

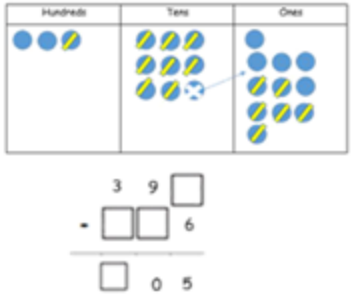
Concrete	Pictorial	Abstract
Column method (using Numicon Pegs) **45 - 26**  Yellow = 10 Red = 1 	Represent the base 10 pictorially 	It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15 
Column method (using place value counters) 234 - 88   	Once the children have had practise with the concrete, they should be able to apply it to any subtraction. Like the other pictorial representations, children represent the counters. 	2 3 4 - 8 8 6

Fluency variation, different ways to ask children to solve 391 - 186:





What's the calculation? What's the answer?



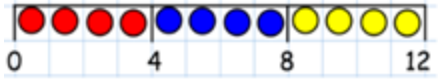
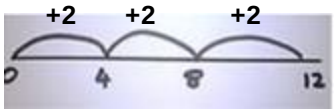
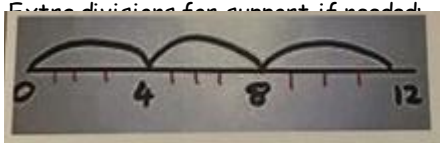
Raj spent £391, Timmy spent £186. How much more did Raj spend?
I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

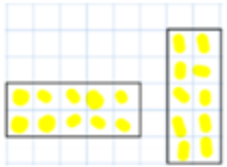
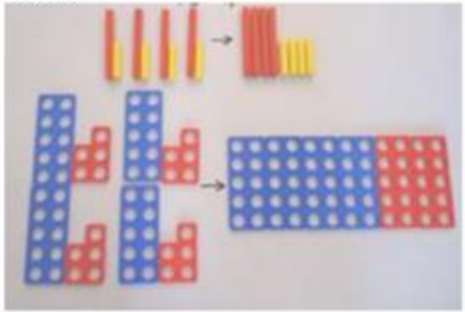
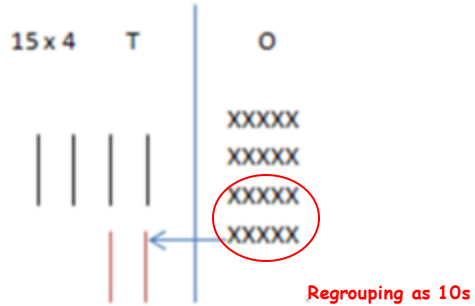
391 - 186 or __ = 391 - 186

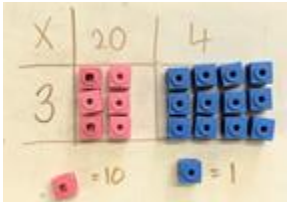
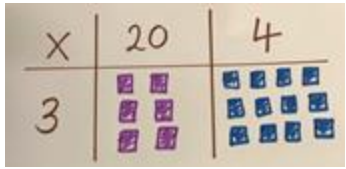
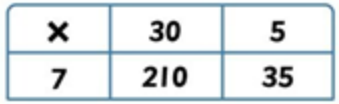
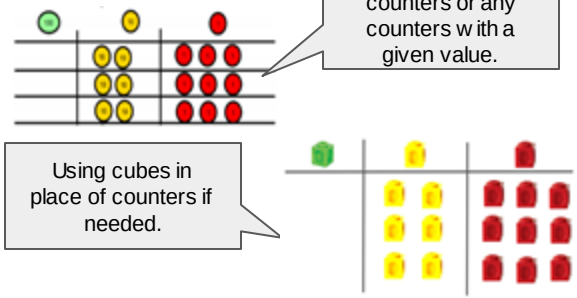
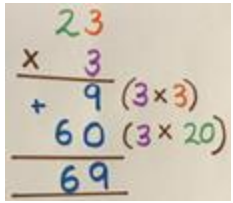
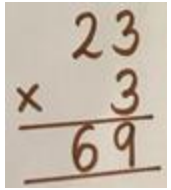
391
-186
____

Find the difference between 391 and 186.
Subtract 186 from 391.
What is 186 less than 391?

Multiplication - **Key vocabulary** which should be used: array, times, multiply, multiplied by, the product of, product, groups of, lots of, is equal to, is the same as, commutativity

Concrete	Pictorial	Abstract
Repeated Addition 3 x 4 or 3 lots of 4 4 + 4 + 4 =  Placing numicon on number lines to show the product.  5 x 3 = 5 + 5 + 5 = 	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method 	4 + 4 + 4 =12 3 x 4 =12
Use **number lines to show repeated groups**.  Opportunity for problem solving (Yr1 NC) 3 children have 4 presents each, how many presents do they have in total? 	**Represent this pictorially** alongside a number line: 	**Abstract number line** 3 x 4 = 12  Extra divisions for support if needed: 

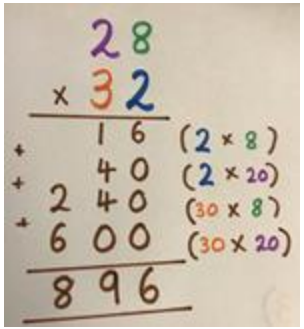
Concrete	Pictorial	Abstract
Use **arrays** to illustrate **commutativity**. (Any objects can be used to create the arrays) **2 x 5 = 5 x 2** 	Children to **draw the arrays**:  Also links to bar model as another demonstration of commutativity.  	Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 =10
Partition to multiply (use numicon, base 10, Cuisenaire rods) 4 x 15 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like:  	**Children to be encouraged to show the steps they have taken through their recording:** 4 x 15 = 4 x 10 = 40 4 x 5 = 20 40 + 20 = 60 (colour coding would support children in understanding where the numbers have come from in the process of calculating)

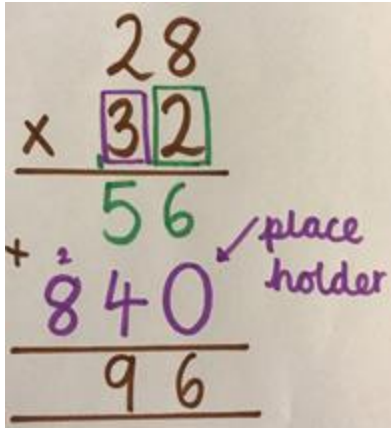
Concrete	Pictorial	Abstract
Grid Method as an extension of partitioning (using counters/ numicon) 24 x 3 =   	**Children to represent their concrete manipulatives in a picture.** Example: (24x3)  	Grid method representing the numbers: 35 x 7 = 245  **210 + 35 = 245**
Formal column method with place value counters or base 10 (at the first stage- no exchanging) 3 x 23  	**Children to represent the counters in a pictorial way**  	Children to record what they are doing to demonstrate understanding.   (Broken down supports understanding of place value moving to the short method - **can be applicable to all stages of this multiplication**)

Advanced Multiplication Methods

When children start to multiply 3d x 3d or 4d x 2d etc, they should be confident with the abstract.
Progression in strategies:

Children breaking down the calculation into steps to aid understanding of place value and regrouping in multiplication:

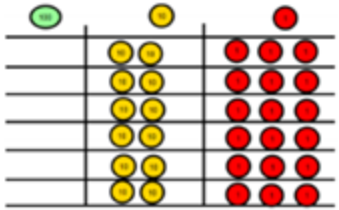




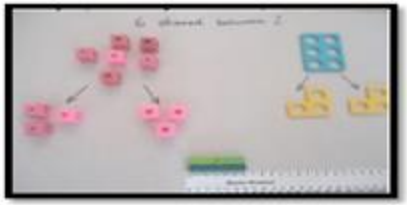
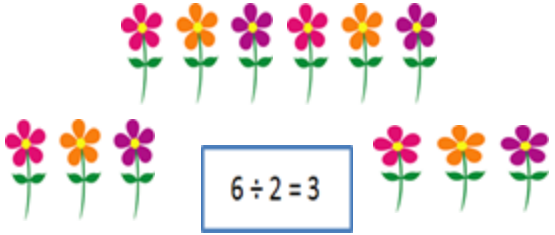
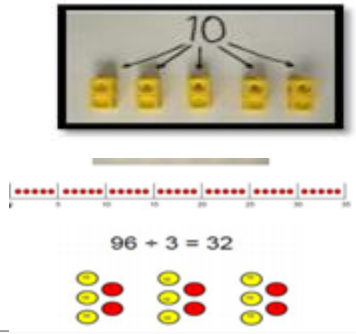
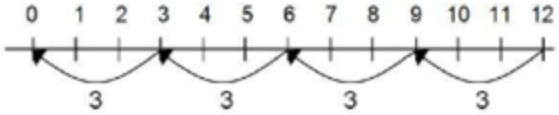
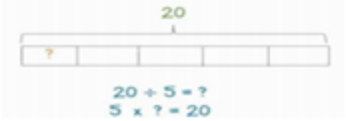
$$\begin{array}{r} 124 \\ \times\ 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ \hline 11 \end{array}$$

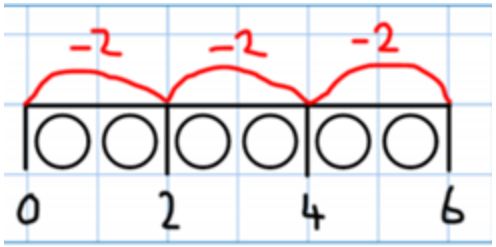
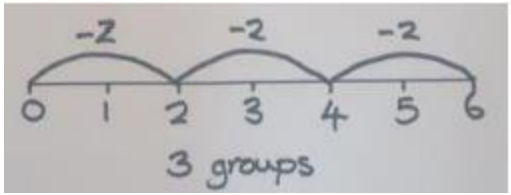
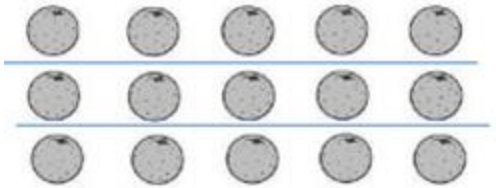
Answer: 3224

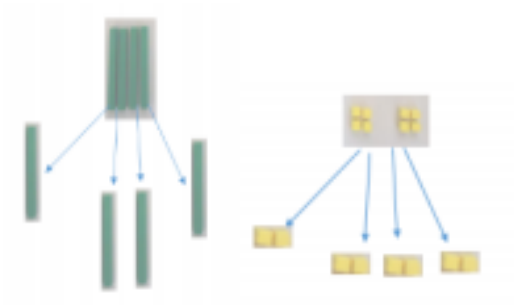
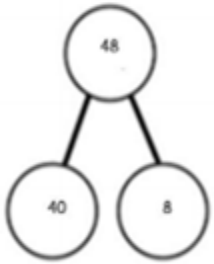
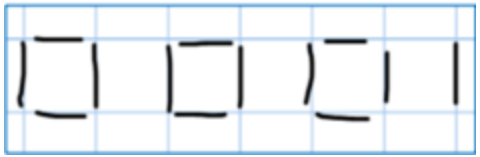
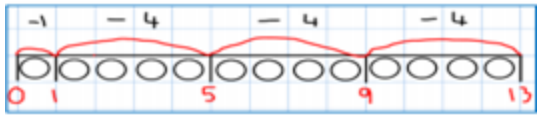
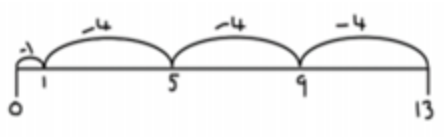
Fluency variation, different ways to ask children to solve 6 x 23:

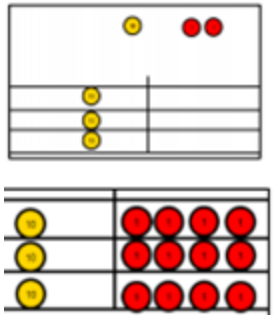
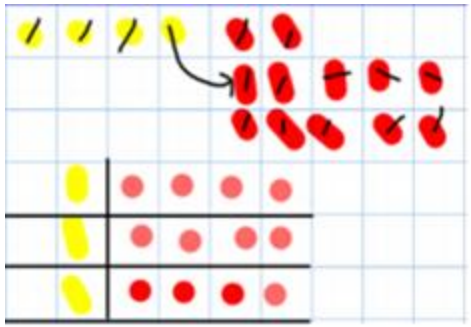
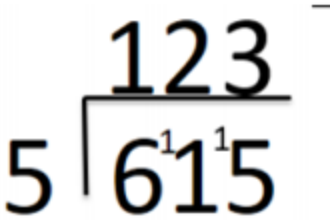
 With the counters, prove that 6 x 23 = 138 Why is 6 x 23 = 32 x 6?	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? Tom saved 23p three days a week. How much did he save in 2 weeks?	Find the product of 6 and 23 6 x 23 = □ = 6 x 23 6 × 23 23 × 6	What's the calculation? What's the answer? 

Division - **Key vocabulary:**: share, group, divide, divided by, half, 'is equal to' 'is the same as'

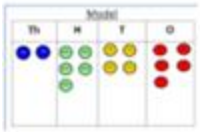
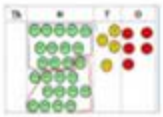
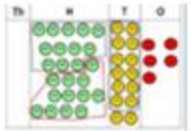
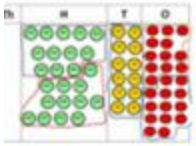
Concrete	Pictorial	Abstract
Sharing objects into groups 6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates) 	Children use pictures or shape to share quantities 	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Concrete	Pictorial	Abstract
Understand division as repeated grouping and subtracting 6 ÷ 2 		Abstract number line 
 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Concrete	Pictorial	Abstract
2d divided by 1d using base 10 (no remainders) SHARING 48 ÷ 4 = 12 Start with the tens	Children to represent the base 10 and sharing pictorially.	
		48 ÷ 4  4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12
2d ÷ 1d with remainders 13 ÷ 4 – 3 remainder 1 	Children to have chance to represent the resources they use in a pictorial way e.g. see below:   	**13 ÷** 

Concrete	Pictorial	Abstract
Sharing using place value counters. 42 ÷ 3= 14  1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10? Exchange the ten for 10 ones and share out 12 ones		42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14
Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- this can also be done using sharing!  Step 1: make 615 Step 2: Circle your groups of 5 Step 3: Exchange 1H for 10T and circle groups of 5 Step 4: exchange 1T for 10 ones and circles groups of 5	This can easily be represented pictorially, till the children no longer need to do it. It can also be done to decimal places if you have a remainder	

Fluency variation, different ways to ask children to solve 615 ÷ 5:			
Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?  	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5 \overline{)615}$ $615 \div 5 =$ $\square = 615 \div 5$ How many 5's go into 615?	What's the calculation? What's the answer?

Concrete	Pictorial	Abstract
 $\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$ 2544 ÷ 12 How many groups of 12 thousands do we have? None  Exchange 2 thousand for 20 hundreds.  $\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$ How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  $\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$ Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.  Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	Children to represent the counters, pictorially and record the subtractions beneath.	$\begin{array}{r} 0 \\ 12\overline{)2544} \end{array}$ Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds. $\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$ Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. $\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$ Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. $\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$ Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.